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## EXERCISES BY DESCRIPTIVE GEOMETRY

EXERCISES BOOK


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Student $\qquad$
Group $\qquad$

Chelyabinsk 2020

## THE COMPLICATED DRAWING OF THE POINT. AXIS AND NON-AXIS METHOD OF MAKING DRAWING

1.1. The Axis Method of Making Drawing.

1. Apply the coordinates of point A (Fig.1). Please, the value of the coordinates (in millimeters) to put in the table. The coefficients of distortion along the axes x and z take equal to 1 , along the axes $\mathrm{y}=0,5$. Construct a complex drawing of point $A$.


Fig. 1

| $\mathrm{X}_{\mathrm{A}}$ (width) |  |
| :--- | :--- |
| $\mathrm{Y}_{\mathrm{A}}$ (depth) |  |
| $\mathrm{Z}_{\mathrm{A}}$ (height) |  |

Write the name of the planes:
$\Pi_{1}$ - $\qquad$
$\Pi_{2}-$
$\Pi_{3}-$


Write the name of the projection of the point $A$ :
$\mathbf{A}_{1}$ - $\qquad$
$\mathbf{A}_{2}$ - $\qquad$
$\mathbf{A}_{3}$ - $\qquad$
2. To construct projections of the points by their coordinates: $A(30,40,25) ; B(20,30,0) ; C(0,20,0)$

3. To construct a complex drawing of points $A(60,10,10)$ и $B(10,30,35)$


Please, write the difference of coordinates:

1. $\mathrm{X}_{\mathrm{A}}-\mathrm{X}_{\mathrm{B}}=$
2. $\mathrm{Y}_{\mathrm{B}}-\mathrm{Y}_{\mathrm{A}}=$
3. $\mathrm{Z}_{\mathrm{B}}-\mathrm{Z}_{\mathrm{A}}=$
4. Construct three projections of the triangle by the coordinates of its vertexes: A(50, 20, 25); $B(20,30,0)$;
C(10, 0, 30)
5. Construct the third projection of the parallelogram KLMN


To write conditions of communication between projections of the point on complicated drawing
1.
2.
3.
$\qquad$
1.2. Non-Axis Method of Making Drawing.


Fig. 2
6. The complex drawing set of interrelate points: $K\left(K_{1}, K_{2}\right) ; L\left(L_{2}, L_{2}\right) ; M\left(M_{1}, M_{2}, M 3\right) ; N\left(N_{1}, N_{2}\right)$; $P\left(P_{2}, P_{3}\right)$. To construct the missing projection points.


## 2. THE COMPLEX DRAWING OF LINE.

7. Construct the complex drawing of the straight line, if the difference of the distances between points $A$ and $B$ from planes of the distances between points $A$ and $B$ from planes of the projections is known. In each case write the name of the line. Show the nature line segment $[A B]$ and angles to the plane of projection for lines which are parallel to projection plane.

## a) $\boldsymbol{\ell}(A, B)$ -

line segment $[A B]$ and angles to the p
b) $h(A, B)-$ $\qquad$ c) $f(A, B)$ - $\qquad$

d) $p(A, B)$ -

f) $i(A, B)$ -

e) $q(A, B)-$

$\mathrm{X}_{\mathrm{A}}-\mathrm{X}_{\mathrm{B}}=0$ мм
$Y_{A}-Y_{B}=0$
$\mathrm{Z}_{\mathrm{B}}-\mathrm{Z}_{\mathrm{A}}=20 \mathrm{MM}$ $\qquad$
g) $g(A, B)-$ $\qquad$


## Relative position of two straight lines

Two straight lines can intersect (have a common point), skew and be parallel. Projections of two parallel line are parallel to each other. Skew lines are not intersected and are not parallel to each other. One of the skew lines can be higher (relation $\Pi_{1}$ ) and further (relation $\Pi_{2}$ )
8. Draw the horizontal $h$ passing the $A$ and crossing the straight line $m$.

9. Draw the frontal f passing the $A$ and skewing the straight line $n$.


Construction starts on the frontal with its horizontal projection. We draw a straight line from point $M$ at right angles to the intersection with the line n . We got $h_{2}$. $h_{1}$ find using the line $1_{1}-1_{2}$
10. Determine the relative position of the line $k, m$, $n$.


The line n and m intersect at point $A$. The line $k$ and $n$ are parallel, because their projections are parallel.
The line $k$ and $m$ are skew lines.
8. Draw the horizontal $h$ and frontal $f$ passing the $A$ and crossing the straight line $m$.

11. Draw the line of general position a passing the $A$ and crossing the straight line $m$.

9. Draw the frontal f passing the $A$ and skewing the straight line $n$.

12. Draw the line of general position a passing the $M$ and skewing the straight line $m$.

10. Draw the straight line $m$ passing the $A$ and would have been parallel to straight line $n$

13. Draw the horizontal $h^{\prime}$, passing the $A$ and would have been parallel to horizontal $h$.


## 3. THE COMPLEX DRAWING OF PLANE

A point and a line belonging to a plane
The point belongs to the plane, since it belongs to one of the lines representing the plane.
The line belongs to a plane if it has a common point with this plane and is parallel to any line of this plane.
The line to a plane if it has two points in common with that plane.
14. Draw the horizontal $h$, frontal $f$ and 15 . Construct the segment $[A B]$, profil line $\rho$, belongs to a plane triangle (ABC) belongs to a plane $\Gamma(a \cap b)$. if the projection points are speci-
 fied.

17. Draw a plane $\boldsymbol{\Sigma}^{\prime}$ through a point $M$ so that it is parallel to the specified plane $\boldsymbol{\Sigma}(A B C)$.

19. Construct the horizontally projection of a triangle $(A B C)$, if it belongs to plane $\Sigma(m \mathrm{II} n)$.

18. Construct the frontal projection of a segment [AB], parallel to the plane $\boldsymbol{\Sigma}(K L M)$.


20 . Is the line $m$ parallel to the plane $\Gamma(a \cap b)$ ?

21. Through the straight line $\boldsymbol{\ell}$ draw: a) horizontal - projecting plane $\boldsymbol{\Sigma}$ at an angle $\boldsymbol{\alpha}=45^{\circ}$ to $\Pi_{2}$

б) frontal - projecting plane $\boldsymbol{\Delta}$.

c) profile - projecting plane $\Gamma$ at an angle $\boldsymbol{\alpha}=30^{\circ}$ to $\Pi_{1}$

22. Draw a plane through the point 0 , given a circle with a diameter $40 \mathrm{~mm}:$ a) horizontal level plane $\boldsymbol{\Sigma}$; б) frontal level plane $\boldsymbol{\Delta}$; в) profile level plane $\Gamma$.

23. Draw the straight line $\boldsymbol{\ell}$, belongs to a plane parallelogram $\Gamma$.

24. To determine the position of points:
a) $A$ and $B$, relative to plane $\Delta$;
б) $\lceil$ and $D$, , relative to plane $\boldsymbol{\Sigma}$.

Write the name of the planes $\Delta$ and $\boldsymbol{\Sigma}$.
A-
$B$ -
$\Delta$ -
c- $\qquad$
D- $\qquad$
4. SURFACES. Face Surfaces. Polyhedrons. Rotation surfaces.
25. Construct the lacking projections of points and lines belonging to the given surfaces (the given projections of point and lines are visible)

1) of the pyramid




## 5. CONSTRUCTION OF THE INTERSECTION LINES OF SURFACES BY THE PROJECTING PLANE

The line of intersection of the surface by the projecting plane is a flat closed line. One projection of the intersection line coincides with the projection of the secant plane within the outline of the intersected surface. The second projection of the intersection line is constructed by points according to the condition that these points belong to a given surface. First of all, control points are defined: points on the edges of polyhedral, extreme and outline

### 5.1. Intersection of a polyhedron by a projecting plane

The intersection line of the polyhedron by the projecting plane is a flat closed polygonal line whose vertices are the points of intersection of the edges, and the sides are the lines of intersection of the faces of the polyhedron with the plane (Fig. 3).

Construct a line of intersection of the pyramid with a horizontally projecting plane.


Her horizontal projections of the reference points 1, 2, 3, 4 are found at the intersection of the edges of the pyramid with the plane 「. The frontal projections of these points are determined using communication lines on the corresponding edges of the pyramid.
Section $2_{2}-3_{2}$ of the broken line on $\Pi_{2}$ is not visible, since it belongs to the nonvisible face $A S B$.

26. Construct intersection lines of these geometric figures with projecting planes. Mark projection of control points. Determine the visibility of the projections of the intersection line and sketches of geometric figures.


### 5.2. Intersection of a surface of rotation by a projecting plane

The intersection line of the rotation surface by the projecting plane is a plane closed curve. To construct this curve, we determine the intersection points of a number of surface generators with a secant plane. The reference points of the line include: extreme (highest, lowest, near, far, left, right), and outline.

Construct a cylinder line of intersection with a frontally projecting plane.


The secant plane is not perpendicular to the axis of rotation of the cylinder. The intersection line is an ellipse (Fig. 4). On the $\Pi_{2}$ plane, the ellipse is projected into the $A_{2} B_{2}$ segment, on the $\Pi_{1}$ plane - into the circle coinciding with the projection of the cylindrical surface; to the plane $\Pi_{3}$ - into an ellipse.

We construct profile projections of points corresponding to an ellipse using two known ones (horizontal and frontal). First of all, we determine the projections of the higher $A$ and lower $B$ points outlined with respect to $\Pi_{3}$ ( $C$ and $D$, then - intermediate, for example, 1 and 2 . Combining the obtained points of the smooth curve with allowance for visibility, we obtain an ellipse, which is a profile projection of a section shape. Points $C$ and $D$ are points of changing visibility to $\Pi_{3}$.

## Construct a line of intersection of the sphere with the frontally projecting plane (Fig. 5)



The plane intersects the sphere in a circle. Depending on the position of the planar projection plane, the environment can be projected into a straight plane, circle or ellipse. The circumference of the section is projected onto the plane A2 in the segment $A_{2} B_{2}$, onto the plane $\Pi_{1}$ in the ellipse, which is built on the points. Points $A$ and $B$ are extreme with respect to $\Pi_{i}: B$ the highest point, $A$ - the lowest. Their frontal projections coincide with the points of intersection of the frontal projection of the plane $\boldsymbol{\Sigma}$ with the outline of the frontal projection of the sphere. Their horizontal projections are found along communication lines on the horizontal projection of the main meridian. We find the frontal projections of points $M$ and $N$ (points of change of visibility with respect to $P_{1}$ ) at the intersection of $\boldsymbol{\Sigma}_{2}$ with the frontal projection of the equator of the sphere. Their horizontal projections are found along the lines of communication on the outline of the horizontal projection of the sphere. Extreme relative to $\Pi_{2}$ points $C$ and $D$ (the closest and the farthest) are determined using the general plane of symmetry $\boldsymbol{\Lambda}$, which is drawn through the center of the sphere perpendicular to the plane $\Sigma$. To find the intermediate points 1 and 2, we use the parallel $n$ passing through these points. The radius of the parallel $R n$, like any other, is measured from the axis to the sketch. On $\Pi_{1}$, the parallel is projected into a circle.

Combining the obtained points of the smooth curve with allowance for visibility, we obtain an ellipse, which is the horizontal projection of the section shape.
27. Construct intersection lines of these geometric figures with projecting planes. Mark the projections of the control points. Determine the visibility of the projections of the intersection line and sketches of geometric figures.


Construct intersection lines of the cone with projecting planes


The plane $\boldsymbol{\Sigma}$ intersects all the generators of the cone. Section line - ellipse. The plane $\Gamma$ is perpendicular to the axis of the cone. Section line - circle


The plane $\boldsymbol{\Delta}$ is parallel to one generatrix of the cone $m(S-1)$. Section line is parabola.

The plane $\Gamma$ passes through the vertex of the cone $S$. The section line - two lines $m(S-1)$ and $n(S-2)$. The plane $\Gamma$ is parallel to two generators $m$ and $n$. Section line - hyperbole.
28. Construct the intersection lines of the cone by the projecting planes. Mark the projections of the control points. Determine the visibility of the projections of the intersection line and sketches of geometric figures.

29. Construct the third projection of the part, the true view of the section " $A-A$ " and its projection.


## 6. THE INTERSECTION OF THE SURFACE WITH A STRAIGHT LINE

Determine the intersection points of a straight line in general position $\boldsymbol{l}$ with the surface of the pyramid $\boldsymbol{\phi}$. Determine the visibility of the projections of the line (Fig. 6).

Depending on the type and relative position of the line and the surface of the points of their intersection, there may be one or more. The basis of their construction is the method of auxiliary surfaces, in accordance with which the construction of the points of intersection of the line $\boldsymbol{\ell}$ and the surface $\boldsymbol{\phi}$ (regardless of their type) is carried out according to the following scheme:

1 Through a given line $\boldsymbol{\ell}$ we draw the auxiliary plane $\boldsymbol{\Sigma}$.
2. We determine the line $m$ of intersection of the auxiliary plane $\Sigma$ and the given surface $\phi$
3. We mark the points $K, L$ of the intersection of the lines $\boldsymbol{l}$ and $m$, which are the desired ones.

In a symbolic notation, the circuit has the form:

1) $\boldsymbol{e} \subset \boldsymbol{\Sigma}$;
2) $\boldsymbol{\Sigma} \cap \boldsymbol{\phi}=m_{i}$
3) $\boldsymbol{\ell} \cap m=K_{1} L$.

The algorithm for solving the problem:

1) $\boldsymbol{e} \subset \boldsymbol{\Sigma} \perp \Pi_{1}$ - through the line $\boldsymbol{e}$ we draw a horizontally projecting plane $\boldsymbol{\Sigma}_{\text {; }}$
2) $\boldsymbol{\phi} \cap \boldsymbol{\Sigma}=m(1,2,3,4)$ - define the line $m(1,2,3,4)$ of intersection of the plane $\boldsymbol{\Sigma}$ and the surface $\phi_{i}$
3) $m(1,2,3,4) \cap \boldsymbol{e}=K_{1} L-$ mark the points $K, L$ of the intersection of the lines $m$ and $\boldsymbol{e}$, which are the desired ones.
Fig. 6
30. Draw the intersection points of lines $a$ and $b$ with given surfaces. Determine the visibility of line projections. Write the algorithm for finding intersection points.

31. $\qquad$ b)

32. 
33. $\qquad$
34. $\qquad$
35. $\qquad$
36. $\qquad$
37. $\qquad$
38. $\qquad$


## 7. LINE OF INTERSECTION OF SURFACES

Two surfaces intersect along a line that simultaneously belongs to each of them.
The intersection line can be straight, broken or curved (flat or spatial)
Solution Scheme:
1.Determine the type and location of the given surfaces relative to each other (inset or penetration) and projection planes (is there a projecting surface)
2.The nature of the intersection line (broken, curve)
3. Determine reference and intermediate points
4. Connect dots and determine visibility.

The intersection of two polyhedral
Sidebar - partly the edges of one and the other polyhedral are involved. The intersection line is one closed plotline
Penetration - all the edges of one polyhedron and part of the edges of another participate.
The intersection line splits into two closed broken lines.
Rule: only segments of vertices that lie in the same face of the first polyhedron and simultaneously in the same face of the second are connected by line segments.

Draw the line of intersection of the pyramid and prism. Define visibility.


1. Given the polyhedral. All edges of the prism intersect the edges of the pyramid. We have a case of penetration. Prism occupies a projecting position on $\mathrm{P}_{3}$.
2. The intersection line broke into two closed broken lines: spatial 1-6-8-9-4-3-1 and flat 2-5-10-7-2. The profile projection of the intersection line coincides with the projection of the prism.
3. The reference points of intersection of the edges of the prism with the faces of the pyramid are determined using horizontal planes of the level $\Gamma$ and $\Gamma^{\prime}$, and the points of intersection of the edges of the pyramid with the faces of the prism - from the condition of belonging.
4. There is no need to define intermediate points.
5. The vertices of the broken line, which belong to one pair of intersecting faces of the pyramid and the prism, are connected by line segments taking into account the visibility Those sections of the broken line that are the intersection line of two projections of the faces of the polyhedral that are visible relative to this plane of the projections are considered visible with respect to a particular projection plane.

Sections $6,-8,-9$, and $7,-10$, of the broken line on 71 are invisible, since they are the result of the intersection of the invisible face of the prism with the surface of the pyramid.
31. Construct the intersection line of the polyhedrons. Determine visibility.
a)




### 7.2. The intersection of a polyhedron and a curved surface

The intersection line is a collection of flat curved lines that are obtained at the intersection of the faces of a polyhedron and a curved surface.
The line of intersection of the prism and the cone. Define visibility.


1) Given a curved surface (cone) and polyhedral (prism). Insert case. Prism occupies a projecting position relative to $\Pi_{3}$.
2) The projection of the intersection line coincides with the profile outline of the prism within the outline of the cone. The intersection line consists of the parts of the ellipse (points $1, C, E$, $\left.A, F, D, 1^{\prime}\right)$, the circle ( $1,4,2,2 \prime 4^{\prime}, 1^{\prime}$ ) and the hyper-balls ( $2,5,3,5^{\prime}, 2^{\prime}$ ), which intersect at points on the edges of the prism ( $1,1^{\prime}, 2$ and $2^{\prime}$ ).
3) Reference points: on the edges of the prism (1, 1', 2 and 2'), the highest and lowest points of the ellipse A and B, points $[$ and $D$ limit the minor axis of the ellipse, outlined $3, E, F, 4,4^{\prime}$.
4) Intermediate points 5 and $5^{\prime}$ for the construction of a hyperbola. All points are found from the condition that they belong to the surface of the cone.
5) We connect the obtained points with smooth curves, taking into account visibility. The ellipse on $\Pi_{2}$ is not visible, since it belongs to the invisible face of the prism.


6) 




### 7.3. The intersection of two curved surface

The intersection line is a spatial curved line.
Sidebar - one closed spatial curve
Penetration - two closed curves.
The order of the line of intersection is equal to the product of the orders of the surfaces.


Draw the line of intersection of the cone and cylinder

1) Defined surface curves. Insert case. The cylinder occupies a projecting position on the frontal plane of projections.
2) The intersection line is a spatial closed curve whose frontal projection coincides with the projection of the cylinder on $\Pi_{2}$ within the outline of the cone.
3) Reference points: $A_{1}\left[\right.$ and $\left[\right.$ are extreme relative to $\Pi_{1}: A$ - the highest point, $C$ and $C^{\prime}$ - the lowest. $D$ and $D^{\prime}$ - outline (points of change of visibility regarding $\Pi_{1}$ ). Points 1, $1^{\prime}$ and 2, 2' - are outlined with respect to $\Pi_{3}$. Points 3 and $3^{\prime}$ - of the tangent point of the generators of the cone and cylinder are also extreme.
4) Intermediate points 4, 4' and 5, 5'. Reference and intermediate points are found according to the principle of belonging to a point on the surface of the cone (using parallels).
5) Connecting the obtained points of the smooth curve taking into account visibility, we obtain a horizontal projection of the intersection line of the given surfaces.
33. Construct intersection lines curved surfaces. Determine the visibility of the projections of the intersection line and sketches of geometric shapes.

7.3.1 Construction of a line of intersection of surfaces by the method of auxiliary secant planes

In the general case, to construct the intersection line of curved surfaces, the method of auxiliary secant planes is used. The position of these planes is chosen so that they intersect with each surface to produce flat sections bounded by circles or straight lines (Fig. 7).


Fig. 7

Construct the projection of the intersection line of the surfaces of the sphere $\boldsymbol{( \Phi )}$ and the torus $(\boldsymbol{\Psi})$. Define visibility.


1) Two surfaces of revolution are given. Case of penetration. There are no projection surfaces.
2) The intersection line - is a spatial closed curve consisting of curves: 1-6-4-8-3-8'-4'-6-1 and 2-7-5-9-3-9'- 5'-7'-2 having a common point 3.
3) Reference points: 1, 2, 3 - extreme, found using the general plane of symmetry $\Lambda_{\text {; }}$ points 4, 4', 5, 5' outlined with respect to $\Pi_{1}$ are defined using the planes $\Sigma$ and $\Sigma$.
4) Intermediate points: 6, 6', 7, 7', 8, 8' (as well as reference points) were found by the algorithm:
5) $\boldsymbol{\Gamma} \cap \boldsymbol{\phi} \wedge \boldsymbol{\Gamma} \cap \boldsymbol{\Psi}, \boldsymbol{\Gamma}| | \Pi_{2 i}$
6) $\Gamma \cap \phi=a$ (окружность),
$\boldsymbol{\Gamma} \cap \boldsymbol{\Psi}=b$ (окружность);
7) $a \cap b=6,7,8,9$.
8) The found points are connected by smooth curves taking into account visibility.
34. Draw the line of intersection of the surfaces. Define visibility.



### 7.3.4. Special cases of intersection of second -order surfaces

THE MONZH THEOREM If two surfaces of the second order are described about the third one or are inscribed into it, their intersection line is disintegrated into two curves of the second order, there planes pass through the line connecting the intersection points of contact lines.

Construct the line of intersection of the cone and cylinder described around the sphere. Define visibility.


1) Given two surfaces of revolution, described around the sphere $\phi$.
2) Based on Monge's theorem, the desired intersection line is split into two second-order flat curves whose planes pass through the straight line $K-L$, connecting the intersection points of the tangent lines $a$ of the sphere $\Phi$ and the cone $\Omega$ and $b$ of the tangent of the sphere $\phi$ and cylinder $\boldsymbol{\Psi}$.
3) Reference points. Extreme (they are also outlined with respect to $\Pi_{2}$ ) points 1 and 2 are constructed using the general plane of symmetry $\Lambda$. Points 3 and $3^{\prime}$ outlined relative to $\Pi_{1}$ are determined from the condition of belonging to the horizontal sketch generators of the cylinder after constructing the projection of the intersection line on $\Pi_{2}$.
4) The intermediate points of the intersection line are found from the condition that their surface belong to the cone $\Omega$ on the corresponding parallels.
5) The found points are connected by a smooth curve taking into account visibility. Points 3 and $3^{\prime}$ on $\Pi_{1}$ are points of change of visibility.
34. Construct the line of intersection of the hemisphere and cone.

35. Find a family of circular sections.

36. Construct the line of intersection

37. Draw intersection lines of given surfaces of revolution. Determine visibility.


## 8. INVOLUTE OF SURFACES

INVOLUTE OF SURFACES is a flat figure obtained by combining the surface with a plane without wrinkles and tears
Build a side scan of the cone and draw point $B$.


Divide the circumference of the base of the cone by a sufficient number of parts (the larger, the more accurate the scan), for example, twelve. We construct the corresponding generators of the cone. We find the generator (S-2), which belongs to point B.

Draw involute of surfaces.
We construct a generator $S$ - 1 , the length of which is equal to the length of the outline generator on $\Pi_{2}$.

From a point $S$ of radius $S-1$ we draw an arc and postpone the length of the chord la twelve times on it. We construct the generator $S$-2. We turn point B around the axis of the cone until it is aligned on the frontal projection with the outline generatrix. We measure the length of the segment $L_{B}$ and, laying it on the generator $S-2$, we get the image of point $B$ on the involutes.
38. Construct a involute of the pyramid

39. Construct a involute of the cylinder and draw points $A$ and $B$ on it that belong to the surface of the cylinder.


## CONSTRUCT A INVOLUTE

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